

REMARKS ON WEAKLY PSEUDOCONVEX BOUNDARIES: ERRATUM

JUDITH BRINKSCHULTE
C. DENSON HILL
MAURO NACINOVICH

It is necessary to make two tiny corrections in [BHN]. This is because the last two authors, having become aware of some inconsistencies in [HN1], have repaired the situation in [HN2], but the results they obtained are slightly different than what was originally claimed. Namely, for local results, it is important to make a distinction between the vanishing of the cohomology of small domains and the validity of the Poincaré lemma. None of the global results in [BHN], concerning weakly pseudoconvex boundaries, require any correction.

The first change needed is that in Theorem 1, part (ii), on page 2, one must add to the hypotheses that x_0 be a *regular point* in the sense of [HN2].

The second change needed is in the example on page 4. The interesting feature of that example (that it is possible to have *vanishing global cohomology*, and at the same time, *infinite dimensional local cohomology*) remains true, but it needs to be re-explained. As a bonus we obtain something new and interesting from it.

Let $z = (z_0, z_1)$ be coordinates in \mathbb{C}^2 , $w = (w_1, \dots, w_{n-1})$ be coordinates in \mathbb{C}^{n-1} . Consider the egg in \mathbb{C}^{n+1} defined by

$$\Omega = \{|z_0|^2 + |z_1|^2 + |w_1|^{2m} + \dots + |w_{n-1}|^{2m} < 1\},$$

for an integer $m \geq 2$. It has a weakly pseudoconvex boundary $\partial\Omega$. For $r = 0, 1, \dots, n-1$, let Σ_{n-r} be the set of points on $\partial\Omega$ at which exactly r components of w are zero. Then $\partial\Omega = \bigcup_{k=1}^n \Sigma_k$, and at each point x_0 of Σ_k , the Levi form of $\partial\Omega$ has k positive and $n-k$ zero eigenvalues. We do not obtain that the Poincaré lemma fails at x_0 in degree (p, k) , as was previously claimed.

1991 *Mathematics Subject Classification.* 32V15, 32V05.

Key words and phrases. weakly pseudoconvex boundaries, tangential Cauchy-Riemann cohomology.

However if $B(x_0, r)$ is any sufficiently small ball centered at $x_0 \in \Sigma_k$ of radius r , in any euclidean metric in \mathbb{C}^{n+1} , it was proved in [HN2] that

$$\dim H^{p,k}(\partial\Omega \cap B(x_0, r)) = +\infty$$

for all $0 \leq p \leq n+1$.

Here is the new observation: Set $U^- = \overline{\Omega} \cap B(x_0, r)$, $U^+ = \mathbb{C}\Omega \cap B(x_0, r)$, and $M = \partial\Omega \cap B(x_0, r)$. By [AH1] we have, for the cohomology of smooth forms on the half open/closed domains U^\pm , that

$$H^{p,k}(M) = H^{p,k}(U^-) \oplus H^{p,k}(U^+).$$

However it follows from [D] or [N] that $\dim H^{p,k}(U^-) = 0$. Thus

$$\dim H^{p,k}(U^+) = +\infty$$

for all $0 \leq p \leq n+1$, which is a new result.

Since $B(x_0, r)$ is sufficiently small, the Levi form of M has at least $k+1$ positive eigenvalues at each point of $M \setminus \Sigma_k$, but only k positive eigenvalues along Σ_k . Note that Σ_k has real codimension $2n - 2k$ in M . Now if Σ_k had been void, we would know from [AH2], that it would be possible to choose the Riemannian metric in \mathbb{C}^{n+1} in such a way as to obtain

$$\dim H^{p,k}(U^+) = 0$$

for all $0 \leq p \leq n+1$. Hence we see that the loss of just one positive eigenvalue along the high codimensional locus Σ_k in M is enough to convert the cohomology of U^+ in degree k from being zero to being infinite dimensional. This was not known before.

REFERENCES

- [AH1] A. Andreotti, C.D. Hill, *E.E. Levi convexity and the Hans Lewy problem I: Reduction to vanishing theorems*, Ann. Sc. Norm. Sup. Pisa **26** (1972), 325-363.
- [AH2] A. Andreotti, C.D. Hill, *E.E. Levi convexity and the Hans Lewy problem II: Vanishing theorems*, Ann. Sc. Norm. Sup. Pisa **26** (1972), 747-806.
- [BHN] J. Brinkschulte, C.D.Hill, M.Nacinovich, *Remarks on weakly pseudoconvex boundaries*, Indag. Mathem. **14** (2003), 1-10.
- [D] A. Dufresnoy, *Sur l'opérateur $\bar{\partial}$ et les fonctions différentiables au sens de Whitney*, Ann. Inst. Fourier **29** (1979), 229-238.
- [HN1] C.D.Hill, M.Nacinovich, *On the failure of the Poincaré lemma for the $\bar{\partial}_M$ -complex*, Math. Ann. **324** (2002), 213-224.
- [HN2] C.D.Hill, M.Nacinovich, *On the failure of the Poincaré lemma for $\bar{\partial}_M$ II*, submitted publication.
- [N] M. Nacinovich, *On boundary Hilbert differential complexes*, Annales Polonici Mathematici **XLVI** (1985), 213-235.